A Multi-Valued Delineation Semantics for Absolute Adjectives

Heather Burnett

University of California, Los Angeles
hburnett@ucla.edu

1   Introduction

This paper provides a novel semantic analysis of the gradability of adjectives of the *absolute* class within a delineation (i.e. comparison-class-based) semantic framework (first presented in [8]). It has been long observed that the syntactic category of bare adjective phrases can be divided into two principle classes: *scalar* (or *gradable*) vs *non-scalar* (*non-gradable*). The principle test for scalarity of an adjective P is the possibility of P to appear (without coercion) in the explicit comparative construction. Thus, we find a first distinction between adjectives like *tall, expensive, straight, empty*, and *dry* on the one hand (ok: *taller, more expensive, straighter, emptier, drier*) and *atomic, pregnant*, and *geographical* on the other (*?more atomic, ?more pregnant, ?more geographical*). It has been argued by many authors that the class of scalar adjectives is further decomposed into two principle subclasses: *relative* adjectives (henceforth *RAs*: ex. *tall, short, expensive, intelligent*) and *absolute* adjectives (henceforth *AAs*: ex. *empty, straight, dry, clean*). Although RAs and AAs behave differently in many syntactic and semantic constructions, the fundamental difference between these two classes of adjectives is generally taken to be that members of the former class have context-sensitive semantic denotations (denotations that vary depending on contextually given comparison classes); whereas, members of the latter class have semantic denotations that are independent of context (cf. [16], [7], [12]). As discussed in [13], this empirical observation raises a puzzle for the delineation approach, since, as will be outlined below, in this framework, the scales associated with adjectival constituents are derived through looking at how their semantic denotations vary across comparison classes. The inability of comparison-class-based frameworks to treat the difference between absolute and relative adjectives has been taken (for example by [6]) to be a major argument against a delineation semantics for scalar adjectives and in favour of a semantics in which degrees and scales are primitives.

In this paper, I present a new solution to the puzzle of the gradability of AAs within the delineation approach, one that takes into account the empirical observation that these constituents can be used imprecisely or vaguely (cf. [10], [11], [6], a.o.). I show that by integrating a simplified version of [8]'s comparison-class-based logical system with the similarity-based multi-valued logical framework proposed by [4] to model the vagueness/imprecision associated with these predi-

cates, we can arrive at new logical framework that can treat the absolute/relative distinction without degrees in the ontology.

The paper is organized as follows: in section 2, I present the delineation framework for the semantic analysis of gradable predicates. Then, in section 3, I present the main ways in which adjectives like *tall* differ from adjectives like *empty*, and I argue that the latter adjectives challenge the comparison-class-based approach. In section 4, I present the empirical observation that absolute adjectives are subject to the phenomenon of vagueness/imprecision, and I introduce the multi-valued logical system that I will be employing to model this phenomenon, [4]'s *Tolerant, Classical, Strict* (TCS). Finally, in section 5, I give my analysis of the gradability of AAs within a delineation extension of TCS. In particular, I propose that the non-trivial scales associated with AAs are derived through looking at comparison-class-based variation in predicate-relative similarity/indifference relations, and I show how these relations can be constructed within this new approach using methods in the same vein as [1] and [14]. The new framework is formally laid out (with the proofs of the main results of the paper) in the appendix.

2   Delineation Semantics

Delineation semantics is a framework for analyzing the semantics of gradable expressions that takes the observation that they are context sensitive to be their key feature. A delineation approach to the semantics of positive and comparative constructions was first proposed by [8], and has been further developed by many authors in the past 30 years. In this framework, scalar adjectives denote sets of individuals and, furthermore, they are evaluated with respect to comparison classes, i.e. subsets of the domain D. The basic idea is that the extension of a gradable predicate can change depending on the set of individuals that it is being compared with. In other words, the semantic denotation of the positive form of the scalar predicate (i.e. *tall*) can be assigned a different set of individuals in different comparison classes.

Definition 1. *CC-relativized interpretation of predicates (informal).*

1. *For a scalar adjective P and a contextually given comparison class $X \subseteq D$,*

   (1) $[\![P]\!]_X \subseteq X.$

2. *For an individual a, a scalar adjective P, and a contextually given comparison class $X \subseteq D$,*

   (2) $[\![a \text{ is } P]\!]_X = \begin{cases} 1 & \text{if } [\![a]\!] \in [\![P]\!]_X \\ 0 & \text{if } [\![a]\!] \in X - [\![P]\!]_X \\ i & \text{otherwise} \end{cases}$

Unlike degree semantics (cf. [6]), delineation semantics takes the positive form as basic and derives the semantics of the comparative form from quantification

over comparison classes. Informally, *John is taller than Mary* is true just in case there is some comparison class with respect to which John counts as tall and Mary counts as not tall.

Definition 2. *Semantics for the comparative (informal)*. *For two individuals a, b and a scalar adjective P, $[\![a \text{ is } P\text{-er than } b]\!] = 1$ iff $a >_P b$, where $>_P$ is defined as:*

(3) $x >_P y$ *iff there is some comparison class X such that $x \in [\![P]\!]_X$ and $y \notin [\![P]\!]_X$.*

As it stands, the analysis of the comparative in definition 2 is very weak and allows some very strange and un-comparative-like relations[1], if we do not say anything about how the extensions of gradable predicates can change in different comparison classes (CCs). A solution to this problem involves imposing some constraints on how predicates like *tall* can be applied in different CCs. In this work, I will adopt the set of constraints on the application of gradable predicates presented in [1] and [2]. Van Benthem proposes three axioms governing the behaviour of individuals across comparison classes. They are the following (presented in my notation):

For $x, y \in D$ and $X \subseteq D$ such that $x \in [\![\mathrm{P}]\!]_X$ and $y \notin [\![\mathrm{P}]\!]_X$,

(4)   **No Reversal (NR:)** There is no $X' \subseteq D$ such that $y \in [\![\mathrm{P}]\!]_{X'}$ and $x \notin [\![\mathrm{P}]\!]_{X'}$.

(5)   **Upward difference (UD)**: For all X', if $X \subseteq X'$, then there is some $z, z' : z \in [\![\mathrm{P}]\!]_{X'}$ and $z' \notin [\![\mathrm{P}]\!]_{X'}$.

(6)   **Downward difference (DD)**: For all X', if $X' \subseteq X$ and $x, y \in X'$, then there is some $z, z' : z \in [\![\mathrm{P}]\!]_{X'}$ and $z' \notin [\![\mathrm{P}]\!]_{X'}$.

No Reversal states that if $x >_P y$, there is no X' such that y is in $[\![P]\!]_{X'}$, but x is not. **Upward Difference** states that if, in the comparison class X, there is a $P/not\ P$ contrast, then a $P/not\ P$ contrast is preserved in every larger CC. Finally, **Downward Difference** says that if in some comparison class X, there is a $P/not\ P$ contrast involving x and y, then there remains a contrast in every smaller CC that contains both x and y. van Benthem shows that these axioms give rise to *strict weak orders*: irreflexive, transitive and almost connected relations[2].

---

[1] For example, suppose in the CC $\{a, b\}$, $a \in [\![P]\!]_{\{a,b\}}$ and $b \notin [\![P]\!]_{\{a,b\}}$. So $a >_P b$. And suppose moreover that, in the larger CC $\{a, b, c\}$, $b \in [\![P]\!]_{\{a,b,c\}}$ and $a \notin [\![P]\!]_{\{a,b,c\}}$. So $b >_P a$. But clearly, natural language comparatives do not work like this: If John is {taller, fatter, wider...} than Mary, Mary cannot also be {taller, fatter, wider...} than John. In other words, $>_P$ must be asymmetric.

[2] The definitions of *irreflexivity, transitivity* and *almost connectedness* are given below.

Definition 3. *Irreflexivity*. *A relation $>$ is irreflexive iff there is no $x \in D$ such that $x > x$.*

Definition 6. **_Strict weak order_**. _A relation $>$ is a strict weak order just in case $>$ is_ **_irreflexive_**, **_transitive_**, _and_ **_almost connected_**.

As discussed in [8], [2] and [14], strict weak orders (also known as _ordinal scales_ in measurement theory) intuitively correspond to the types of relations expressed by many kinds of comparative constructions[3]. Thus, the theorem in 1 is an important result in the semantic analysis of comparatives, and it shows that scales associated with gradable predicates can be constructed from the context-sensitivity of the positive form and certain axioms governing the application of the predicate across different contexts.

Theorem 1. **_Strict Weak Order_**. _For all P, $>_P$ is a strict weak order._

Proof. [1]; [2], p. 116.   □

This analysis seems appropriate for relative predicates like _tall_ and _short_; however, as we will see in the next section, it does not capture the certain aspects of the meaning of absolute predicates like _empty_ and _straight_.

3   The Absolute/Relative Distinction

Following many authors, I take the principle way in which AAs like _empty_ and _straight_ differ from RAs like _tall_ and _fat_ is that AAs are not context-sensitive in the same way that RAs are. One test that shows this is the _definite description_ test. As observed by [6] and [15] a.o., adjectives like _tall_ and _empty_ differ in whether they can 'shift' their thresholds (i.e. criteria of application) to distinguish between two individuals in a two-element comparison class when they appear in a definite description. For example, suppose there are two containers (A and B), and neither of them are particularly tall; however, A is (noticeably) taller than B. In this situation, if someone asks me (7-a), then it is very clear that I should pass A. Now suppose that container A has less liquid than container B, but neither container is particularly close to being completely empty. In this situation, unlike what we saw with _tall_, (7-b) is infelicitous.

(7)   a.   Pass me **the tall one**.

Definition 4. **_Transitivity._** _A relation $>$ is transitive iff for all $x, y, z \in D$, if $x > y$ and $y > z$, then $x > z$._

Definition 5. **_Almost Connectedness_**. _A relation $>$ is almost connected iff for all $x, y \in D$, if $x > y$, then for all $z \in D$, either $x > z$ or $z > y$._

[3] For example, one cannot be taller than oneself; therefore $>_{tall}$ should be irreflexive. Also, if John is taller than Mary, and Mary is taller than Peter, then we know that John is also taller than Peter. So $>_{tall}$ should be transitive. Finally, suppose John is taller than Mary. Now consider Peter. Either Peter is taller than Mary (same height as John or taller) or he is shorter than John (same height as Mary or shorter). Therefore, $>_{tall}$ should be almost connected.

b.   Pass me **the empty one**.

In other words, unlike RAs, AAs cannot change their criteria of application to distinguish between objects that lie in the middle of their associated scale. Using this test, we can now make the argument that adjectives like *full, straight,* and *bald* are absolute, since (8-a) is infelicitous if neither object is (close to) completely full/straight/bald. Likewise, we can make the argument that *dirty, wet*, and *bent* are also absolute, since (8-b) is infelicitous when comparing two objects that are at the middle of the dirtiness/wetness/curvature scale (i.e. both of them are dirty/wet/bent).

(8)   **Absolute Adjectives**
     a.   Pass me the **full/straight/bald** one.
     b.   Pass me the **dirty/wet/bent** one.

Furthermore, we can make the argument that *long, expensive,* and even colour adjectives like *blue* are relative, since the (9) is felicitous when comparing two objects when both or neither are particularly long/expensive/blue[4].

(9)   **Relative Adjectives**
     Pass me the **long/expensive/blue** one.

How can we capture this distinction in a delineation framework? An idea that has been present in the literature for a long time, and has recently been incarnated in, for example, [7] and [12], is that unlike *tall* or *long* that have a context sensitive meaning, adjectives like *straight, empty* or *bald* are not context sensitive (hence the term *absolute* adjective). That is, in order to know who the bald people are or which rooms are empty, we do not need compare them to a certain group of other individuals, we just need to look at their properties. To incorporate this idea into the delineation approach, I propose (following an idea in [14]) that, in a semantic framework based on comparison classes, what it means to be non-context-sensitive is to have your denotation be invariant across classes. Thus, for an absolute adjective Q and a comparison class X, it suffices to look at what the extension of Q is in the maximal CC, the domain D, in order to know what $[\![Q]\!]_X$ is. I therefore propose that a different axiom set governs the semantic interpretation of the members of the absolute class that does not apply to the relative class: the singleton set containing the *absolute adjective axiom*.

(10)   **Absolute Adjective Axiom (AAA).**
     If Q $\in AA$, then for all $X \subset D$ and $x \in X$, $x \in [\![Q]\!]_X$ iff $x \in [\![Q]\!]_D$.

In other words, the semantic denotation of an absolute adjective is set with respect to the total domain, and then, by the AAA, the interpretation of Q in D is replicated in each smaller comparison class. The AAA is very powerful: as shown by theorem 2, the scales that the semantic denotations of absolute

---

[4] For an example of the use of a colour adjective like *blue* to distinguish between two not particularly blue objects, see [5].

constituents give rise to are very small, essentially trivial. In particular, the relations denoted by the absolute and non-scalar comparative ($>_Q$) do not allow for the predicate to distinguish three distinct individuals.

Theorem 2. *If $Q \in AA$, then there is no CC model M such that, for distinct $x, y, z \in D$, $x >_Q y >_Q z$.*

Proof. Let $Q \in AA$ (so it satisfies the AAA). Suppose for a contradiction that there is some CC model $M = \langle D, CC, \llbracket \cdot \rrbracket \rangle$ such that x, y, z are distinct members of D, and $x >_Q y >_Q z$. Then, by definition 2, there is some $X \in CC$ such that $x \in \llbracket Q \rrbracket_X$ and $y \notin \llbracket Q \rrbracket_X$. Therefore, by the AAA, $y \notin \llbracket Q \rrbracket_D$. Furthermore, since $y >_Q z$, there is some $X' \in CC$ such that $y \in \llbracket Q \rrbracket_{X'}$ and $z \notin \llbracket Q \rrbracket_{X'}$. Since $y \in \llbracket Q \rrbracket_{X'}$, by the AAA, $y \in \llbracket Q \rrbracket_D$. $\bot$ So there is no CC model M such that, for distinct $x, y, z \in D$, $x >_Q y >_Q z$. $\qquad \square$

Absolute adjectives thus raise a puzzle for delineation analyses:

(11)   **The Puzzle of Absolute Adjectives:**
         If AAs have non-context-sensitive semantic denotations, how can they be gradable?

In the rest of the paper, I will provide a solution to this puzzle.

4   Vagueness/Imprecision with AAs

Of course, saying that adjectives like *empty*, *bald*, and *straight* are not at all context-sensitive is clearly false. As observed by very many authors (ex. [16], [10], [11], [7], [6] a.o.), the criteria for applying an absolute adjective can vary depending on context, as exemplified in (12).

(12)   a.   Only two people came to opening night; the theatre was empty.
         b.   Two people didn't evacuate; the theatre wasn't empty when they started fumigating.

Rather than being attributed directly to the context-sensitivity of their semantic denotation, the contextual variation in the application of absolute predicates is generally attributed to something that is variably called "imprecision", "loose talk" or "vagueness", among other things. I therefore propose that the context-sensitivity that allows for the construction of non-trivial scales is not semantic, as in the case of relative adjectives (as outlined in section 2), but **pragmatic**: although the semantic denotation of an absolute predicate does not vary across comparison classes, its denotation on its imprecise use does.

As mentioned in the introduction, the approach that I will adopt to model the effects of vagueness/imprecision is [4]'s *Tolerant, Classical, Strict* (TCS). This system was developed as a way to preserve the intuition that vague and imprecise

predicates[5] are *tolerant* (i.e. satisfy $\forall x \forall y [P(x) \mathbin{\&} x \sim_P y \rightarrow P(y)]$, where $\sim_P$ is a 'little by little' or indifference relation for a predicate P), without running into the Sorites paradox[6]. [4] adopt a non-classical logical framework with three notions of satisfaction: classical satisfaction, tolerant satisfaction, and its dual, strict satisfaction. Formulas are tolerantly/strictly satisfied based on classical truth and predicate-relative, possibly non-transitive *indifference relations*. For a given predicate P, an indifference relation, $\sim_P$, relates those individuals that are viewed as sufficiently similar with respect to P. For example, for the predicate *empty*, $\sim_{empty}$ would be something like the relation "differ by a number of objects that is irrelevant for our purposes/contain roughly the same number of objects". Since these relations are given by context, we assume that they are part of the model. I give the definition of the indifference relations (within a comparison class-based framework) below.

Definition 7. *CC-relativized indifference relations.* *For all scalar adjectives P and comparison classes $X \subseteq D$,*

(13) $\sim_P^X$ *is a binary relation on the elements of X that is reflexive and symmetric (but not necessarily transitive).*

In this framework, we say that *Room A is empty* is tolerantly true just in case Room A contains a number of objects that do not cause us to make a distinction between it and a completely empty room in the context. For the purposes of the analyses in this paper, I will suppose that classical satisfaction and classical denotations correspond to regular semantic satisfaction and semantic denotations, while tolerant and strict satisfaction and denotations correspond to pragmatic notions[7]. The three notions of satisfaction are defined within a comparison-class-based system[8] as shown below.

Definition 8. *Classical ($\llbracket \rrbracket^c$), tolerant ($\llbracket \rrbracket^t$), and strict ($\llbracket \rrbracket^s$) interpretation of predicates.* *For all scalar adjectives P and $X \subseteq D$,*

1. $\llbracket P \rrbracket_X^c \subseteq X$.
2. $\llbracket P \rrbracket_X^t = \{x : \exists d \sim_P^X x \text{ and } d \in \llbracket P \rrbracket_X^c\}$.

---

[5] The system in [4] was proposed to model the puzzling properties of vague language with relative predicates like *tall*; however, I suggest that the results in this paper show that it has a natural application to modelling similar effects with absolute adjectives.

[6] Note that on their imprecise use, absolute predicates like *bald* and *empty* give rise to Soritical-type reasoning: how many hairs must someone have before they stop being considered bald? How many seats must be filled before a theatre is no longer considered empty?

[7] As such, my interpretation of the framework bares many similarities with [9]'s *Pragmatic Halos* approach to modelling "pragmatic slack" or "loose talk".

[8] Note that, in his 1980 paper, Klein adopts a supervaluationist account of the vagueness of scalar adjectives. Thus, the integration of Klein's basic semantics for the comparative construction with a similarity-based account of vagueness is a departure from the system presented in [8].

3. $[\![P]\!]^s_X = \{x : \forall d \sim^X_P x, d \in [\![P]\!]^c_X\}$.

Definition 9. ***Classical, tolerant, and strict satisfaction.*** *For all individuals a, scalar predicates P, and comparison classes $X \subseteq D$,*

1. $[\![a \text{ is } P]\!]^c_X = \begin{cases} 1 & \text{if } [\![a]\!] \in [\![P]\!]^c_X \\ 0 & \text{if } [\![a]\!] \in X - [\![P]\!]^c_X \\ i & \text{otherwise} \end{cases}$

2. $[\![a \text{ is } P]\!]^t_X = \begin{cases} 1 & \text{if } [\![a]\!] \in [\![P]\!]^t_X \\ 0 & \text{if } [\![a]\!] \in X - [\![P]\!]^t_X \\ i & \text{otherwise} \end{cases}$

3. $[\![a \text{ is } P]\!]^s_X = \begin{cases} 1 & \text{if } [\![a]\!] \in [\![P]\!]^s_X \\ 0 & \text{if } [\![a]\!] \in X - [\![P]\!]^s_X \\ i & \text{otherwise} \end{cases}$

The definitions of the tolerant and strict comparative relations are parallel to the classical comparative (definition 2).

Definition 10. ***Classical/tolerant/strict comparative (informal).*** *For two individuals a, b and a scalar adjective P, $[\![a \text{ is } P\text{-er than } b]\!]^{t/c/s} = 1$ iff $a >^{t/c/s}_P b$, where $>^{t/c/s}_P$ is defined as:*

(14) $x >^{t/c/s}_P y$ *iff there is some comparison class X such that $x \in [\![P]\!]^{t/c/s}_X$ and $y \notin [\![P]\!]^{t/c/s}_X$.*

The precise definition of TCS, set within a comparison-class-based approach to the semantics of scalar terms, is given in the appendix.

5   Analysis of Absolute Adjectives

In order to account for how AAs can have, at the same time, a semantic denotation that is constant across CCs, but at the same time be associated with non-trivial scales, I propose that what can vary across CCs are the indifference relations i.e., the $\sim^X_Q$s. For example, if I compare Homer Simpson, who has exactly two hairs, directly with Yul Brynner (who has zero hairs), the two would not be considered indifferent with respect to baldness (Homer has hair!). However, if I add Marge Simpson into the comparison class (she has a very large hairdo), then Yul and Homer start looking much more similar, when it comes to baldness. Thus, I propose, it should be possible to order individuals with respect to how close to being completely bald (or empty or straight) they are by looking at in which comparison classes they are considered indifferent to completely bald/empty/straight individuals[9].

---

[9] The idea is conceptually similar in some sense (although extremely different in its execution) to a suggestion made by [12], with respect to how an adjective like *empty* can be both absolute and gradable.

In what follows, I present a set of axioms that constrain indifference relations between individuals across comparison classes. Recall that I proposed that, unlike relative adjectives which are only subject to van Benthem's axioms (NR, UD, and DD), absolute adjectives are subject to the AAA. Then, in the spirit of [1] and [13], I will show that these axioms will allow us to construct non-trivial strict weak orders from the tolerant meaning of absolute predicates[10].

5.1 Pragmatic Axiom Set

I propose the following axioms to constrain indifference relations[11].

(15) **Tolerant No Skipping (T-NS):** For an AA Q, $X \in \mathcal{P}(D)$ and $x, y \in X$, if $x \sim_Q^X y$ and there is some $z \in X$ such that $x \geq_Q^t z \geq_Q^t y$, then $x \sim_Q^X z$.

Tolerant No Skipping says that, if person A is indistinguishable from person B, and there's a person C lying in between persons A and B on the relevant tolerant scale, then A and C (the greater two of $\{A, B, C\}$) are also indistinguishable. As discussed in the appendix, T-NS performs a very similar function to van Benthem's *No Reversal*.

We now have two axioms that talk about how indifference relations can change across comparison classes. I call these the *granularity* axioms.

(16) **Granularity 1 (G1):** For an AA Q, $X \in \mathcal{P}(D)$, and $x, y \in X$, if $x \sim_Q^X y$, then for all $X' \subseteq D : X \subseteq X'$, $x \sim_Q^{X'} y$.

G1 says that if person A and person B are indistinguishable in comparison class X, then they are indistinguishable in all supersets of X. This is meant to

---

[10] For lack of space, I will only address the analysis of so-called *total* or *universal* AAs like *empty, bald*, and *straight*. However, the analysis of partial/existential AAs like *dirty* and *wet* is essentially the dual of the analysis of total AAs, with non-trivial scales being constructed out of strict denotations instead of tolerant ones. See [3] for discussion.

[11] One of the axioms (T-NS) makes reference to a 'tolerantly greater than or equal relation' ($\geq_Q^t$): We first define an equivalence relation $\approx_P$:

Definition 11. *Tolerantly Equivalent.* ($\approx^t$) *For a predicate Q and $a, b \in D$,*

(i) $a \approx_Q^t b$ iff $a \not\succ_Q^t b$ and $b \not\succ_Q^t a$.

Now we define $\geq^t$:

Definition 12. *Tolerantly greater than or equal.* ($\geq^t$) *For $a, b \in D$,*

(ii) $a \geq_Q^t b$ iff $a >_Q^t b$ or $a \approx_Q^t b$.

reflect the fact that the larger the domain is (i.e. the larger the comparison class is), the more things can cluster together[12].

(17)   **Granularity 2 (G2):** For an AA Q, $X, X' \subseteq D$, and $x, y \in X$, if $X \subset X'$ and $x \not\prec_Q^X y$ and $x \sim_Q^{X'} y$, then $\exists z \in X' - X : x \not\prec_Q^{X'} z$.

G2 says that, if person A and person B are distinguishable in one CC, X, and then there's another CC, X', in which they are indistinguishable, then there is some person C in X'-X that is distinguishable from person A. This axiom is similar in spirit to van Benthem's *Upward Difference* in that it ensures that, if there is a contrast/distinction in one comparison class, the existence of contrast is maintained in all the larger CCs.

The final axiom that we need is *Minimal Difference:*

(18)   **Minimal Difference (MD):** For an AA Q and $x, y \in D$, if $x >_Q^c y$, then $x \not\prec_Q^{\{x,y\}} y$.

Minimal Difference says that, if, at the finest level of granularity, you would make a classical distinction between two individuals, then they are not indistinguishable at that level of granularity. MD is similar in spirit to van Benthem's *Downward Difference* because it allows us to preserve contrasts down to the smallest comparison classes.

With these axioms, we can prove the main result of the paper (which is proved in the appendix):

(19)   **Theorem 3**. If Q is an absolute adjective, then $>_Q^t$ is a strict weak order.

6   Conclusion

In this paper, I gave a new analysis of the semantics and pragmatics of absolute adjectives, and, in particular, I addressed the question of how AAs can have a non-context-sensitive semantic denotation but still be gradable with a delineation framework. I showed that the scales (i.e. strict weak orders) that are associated with absolute predicates can be derived in within the multi-valued delineation TCS system from certain intuitive statements about how individuals can and cannot be indifferent across comparison classes. Thus, I argue that the puzzles raised by absolute adjectives for the delineation approach can be solved, provided that we have an appropriate framework to treat vagueness and imprecision.

References

1. J. van Benthem. (1982). "Later than late: On the logical origin of the temporal order". *Pacific Philosophical Quarterly*, 63:193203.

---

[12] G1 can be weakened a bit to allow some indifference relations to be undone in larger CCs. In this case, we derive semi-orders instead of strict weak orders (cf. [3]).

2. J. van Benthem. (1990). *The Logic of Time*. Dordrecht: Reidel.

3. H. Burnett. (2012). *The Grammar of Tolerance: On Vagueness, Context-Sensitivity, and the Origin of Scale Structure*. PhD Dissertation. UCLA.

4. P. Cobreros, P. Égré, D. Ripley, and R. van Rooij. (2011). "Tolerant, Classical, Strict." *Journal of Philosophical Logic*. (forthcoming).

5. D. Fara. (2000). "Shifting Sands: An interest-relative theory of vagueness." *Philosophical Topics*. 20: 45-81.

6. C. Kennedy. (2007). "Vagueness and Grammar: The study of relative and absolute gradable predicates." *Linguistics and Philosophy*. 30: 1-45.

7. C. Kennedy and L. McNally. (2005). "Scale structure and the semantic typology of gradable predicates". *Language*. 81:345-381.

8. E. Klein. (1980). "A semantics for positive and comparative adjectives". *Linguistics and Philosophy*. 4:1-45.

9. P. Lasersohn. (1999). "Pragmatic Halos." *Language*. 75: 522-571.

10. D. Lewis. (1979). "Score-keeping in a language game". *Journal of Philosophical Logic*, 8: 339-359.

11. M. Pinkal. (1995). *Logic and Lexicon*. Dordrecht: Kluwer Academic Publishers.

12. F. Récanati. (2010). *Truth-Conditional Pragmatics*. Oxford: OUP.

13. R. van Rooij. (2011). "Vagueness and Linguistics". In G Ronzitti, editor, *The vagueness handbook*, Dordrecht: Springer. pp. 1-57.

14. R. van Rooij. (2011). "Implicit vs explicit comparatives". In Paul Egré and Nathan Klinedinst (eds.), *Vagueness and Language Use*, Palgrave Macmillan.

15. K. Syrett et al. (2010). "Meaning and context in children's understanding of gradable adjectives". *Journal of Semantics*, 27:1-35.

16. P. Unger. (1975). *Ignorance*. Oxford: Clarendon Press.

7   Appendix: Framework and Proofs

7.1   The Framework: Delineation TCS

Language

Definition 13. *Vocabulary*. *The vocabulary consists of the following expressions:*

1. *A series of individual constants: $a_1, a_2, a_3 \ldots$*
2. *A series of individual variables: $x_1, x_2, x_3 \ldots$*
3. *Two series of unary predicate symbols:*
   - *Relative scalar adjectives: $P_1, P_2, P_3 \ldots$*
   - *Absolute scalar adjectives: $Q_1, Q_2, Q_3 \ldots$*
4. *For every unary predicate symbol P, there is a binary predicate $>_P$.*
5. *Quantifiers and connectives $\forall$, $\vee$ and $\neg$, plus parentheses.*

Definition 14. *Syntax*.

1. *Variables and constants (and nothing else) are terms.*
2. *If t is a term and P is a predicate symbol, then $P(t)$ is a well-formed formula (wff).*
3. *If t_1 and t_2 are terms and P is a predicate symbol, then $t_1 >_P t_2$ is a wff.*
4. *For any variable x, if ϕ and ψ are wffs, then $\neg\phi$, $\phi \vee \psi$, and $\forall x \phi$ are wffs.*
5. *Nothing else is a wff.*

Semantics

Definition 15. ***C(lassical)-model***. *A c-model is a tuple* $M = \langle D, m \rangle$ *where* D *is a non-empty domain of individuals, and* m *is a function from pairs consisting of a member of the non-logical vocabulary and a comparison class (a subset of the domain) satisfying:*

  − *For each individual constant* a_1, $m(a_1) \in D$.
  − *For each* $X \in \mathcal{P}(D)$ *and for each predicate* P, $m(P, X) \subseteq X$.

Definition 16. ***T(olerant)-model***. *A t-model is a tuple* $M = \langle D, m, \sim \rangle$, *where* $\langle D, m \rangle$ *is a model and* $\sim$ *is a function from predicate/comparison class pairs such that:*

  − *For all* P *and all* $X \in \mathcal{P}(D)$, $\sim_P^X$ *is a binary relation on* X *that is reflexive, symmetric, but not necessarily transitive.*

Definition 17. ***Assignment***. *An assignment for a c/t-model* M *is a function* $g : \{x_n : n \in \mathbb{N}\} \to D$ *(from the set of variables to the domain* D*).*

Definition 18. ***Interpretation***. *An interpretation* $[\![\cdot]\!]_{M,g}$ *is a pair* $\langle M, g \rangle$, *where* M *is a t-model, and* g *is an assignment.*

Definition 19. ***Interpretation of terms*** *(*$[\![\cdot]\!]_{M,g}$*). For a model* M, *an assignment* g,

  1. If x_1 *is a variable,* $[\![x_1]\!]_{M,g} = g(x_1)$.
  2. If a_1 *is a constant,* $[\![a_1]\!]_{M,g} = m(a_1)$.

    In what follows, for an interpretation $[\![\cdot]\!]_{M,g}$, a variable x_1, and a constant a_1, let $g[a_1/x_1]$ be the assignment for M which maps x_1 to a_1, but agrees with g on all variables that are distinct from x_1.

Definition 20. ***Classical Satisfaction*** *(*$[\![\cdot]\!]^c$*). For all interpretations* $[\![\cdot]\!]_{M,g}$, *all* $X \in \mathcal{P}(D)$, *all formulas* ϕ, ψ, *all predicates* P, *and all terms* t_1, t_2,

1. $[\![P(t_1)]\!]^c_{M,g,X} = \begin{cases} 1 & \text{if } [\![t_1]\!]_{M,g} \in m(P, X) \\ 0 & \text{if } [\![t_1]\!]_{M,g} \in X - m(P, X) \\ i & \text{otherwise} \end{cases}$

2. $[\![t_1 >_P t_2]\!]^c_{M,g,X} = \begin{cases} 1 & \text{if there is some } X' \subseteq D : [\![P(t_1)]\!]^c_{M,g,X'} = 1 \text{ and } [\![P(t_2)]\!]^c_{M,g,X'} = 0 \\ 0 & \text{otherwise} \end{cases}$

3. $[\![\neg \phi]\!]^c_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]^c_{M,g,X} = 0 \\ 0 & \text{if } [\![\phi]\!]^c_{M,g,X} = 1 \\ i & \text{otherwise} \end{cases}$

4. $[\![\phi \vee \psi]\!]^c_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]^c_{M,g,X} = 1 \text{ or } [\![\psi]\!]^c_{M,g,X} = 1 \\ 0 & \text{if } [\![\phi]\!]^c_{M,g,X} = [\![\psi]\!]^c_{M,g,X} = 0 \\ i & \text{otherwise} \end{cases}$

5. $[\![\forall x_1 \phi]\!]^c_{M,g,X} = \begin{cases} 1 & \text{if for every } a_1 \in X, [\![\phi]\!]^c_{M,g[a_1/x_1],X} = 1 \\ 0 & \text{if for some } a_1 \in X, [\![\phi]\!]^c_{M,g[a_1/x_1],X} = 0 \\ i & \text{otherwise} \end{cases}$

Definition 21. Tolerant Satisfaction($[\![\cdot]\!]^t$). *For all interpretations $[\![\cdot]\!]_{M,g}$, all $X \in \mathcal{P}(D)$, all formulas ϕ, ψ, all predicates P, and all terms t_1, t_2,*

1. $[\![P(t_1)]\!]^t_{M,g,X} = \begin{cases} 1 & \text{if there is some } a_1 \sim^X_P [\![t_1]\!]_{M,g} : [\![P(a_1)]\!]^c_{M,g,X} = 1 \\ 0 & \text{if } [\![t_1]\!]_{M,g} \in X, \text{and there is no } a_1 \in X : a_1 \sim^X_P [\![t_1]\!]_{M,g} \\ i & \text{otherwise} \end{cases}$

2. $[\![t_1 >_P t_2]\!]^t_{M,g,X} = \begin{cases} 1 & \text{if there is some } X' \subseteq D : [\![P(t_1)]\!]^t_{M,g,X'} = 1 \text{ and } [\![P(t_2)]\!]^t_{M,g,X'} = 0 \\ 0 & \text{otherwise} \end{cases}$

3. $[\![\neg \phi]\!]^t_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]^s_{M,g,X} = 0 \\ 0 & \text{if } [\![\phi]\!]^s_{M,g,X} = 1 \\ i & \text{otherwise} \end{cases}$

4. $[\![\phi \vee \psi]\!]^t_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]^t_{M,g,X} = 1 \text{ or } [\![\psi]\!]^t_{M,g,X} = 1 \\ 0 & \text{if } [\![\phi]\!]^t_{M,g,X} = [\![\psi]\!]^t_{M,g,X} = 0 \\ i & \text{otherwise} \end{cases}$

5. $[\![\forall x_1 \phi]\!]^t_{M,g,X} = \begin{cases} 1 & \text{if for every } a_1 \in X, [\![\phi]\!]^t_{M,g[a_1/x_1],X} = 1 \\ 0 & \text{if for some } a_1 \in X, [\![\phi]\!]^t_{M,g[a_1/x_1],X} = 0 \\ i & \text{otherwise} \end{cases}$

Definition 22. Strict Satisfaction($[\![]\!]^s$). *For all interpretations $[\![\cdot]\!]_{M,g}$, all $X \in \mathcal{P}(D)$, all formulas ϕ, ψ, all predicates P, and all terms t_1, t_2,*

1. $[\![P(t_1)]\!]^s_{M,g,X} = \begin{cases} 1 & \text{if for all } a_1 \sim^X_P [\![t_1]\!]_{M,g} : [\![P(a_1)]\!]^c_{M,g,X} = 1 \\ 0 & \text{if } [\![t_1]\!]_{M,g} \in X, \text{and there is no } a_1 \in X : a_1 \sim^X_P [\![t_1]\!]_{M,g} \\ i & \text{otherwise} \end{cases}$

2. $[\![t_1 >_P t_2]\!]^s_{M,g,X} = \begin{cases} 1 & \text{if there is some } X' \subseteq D : [\![P(t_1)]\!]^s_{M,g,X'} = 1 \text{ and } [\![P(t_2)]\!]^s_{M,g,X'} = 0 \\ 0 & \text{otherwise} \end{cases}$

3. $[\![\neg \phi]\!]^s_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]^t_{M,g,X} = 0 \\ 0 & \text{if } [\![\phi]\!]^t_{M,g,X} = 1 \\ i & \text{otherwise} \end{cases}$

4. $[\![\phi \vee \psi]\!]^s_{M,g,X} = \begin{cases} 1 & \text{if } [\![\phi]\!]^s_{M,g,X} = 1 \text{ or } [\![\psi]\!]^s_{M,g,X} = 1 \\ 0 & \text{if } [\![\phi]\!]^t_{M,g,X} = [\![\psi]\!]^t_{M,g,X} = 0 \\ i & \text{otherwise} \end{cases}$

5. $[\![\forall x_1 \phi]\!]^s_{M,g,X} = \begin{cases} 1 & \text{if for every } a_1 \in X, [\![\phi]\!]^s_{M,g[a_1/x_1],X} = 1 \\ 0 & \text{if for some } a_1 \in X, [\![\phi]\!]^s_{M,g[a_1/x_1],X} = 0 \\ i & \text{otherwise} \end{cases}$

Proposed Axioms for AAs [13]

(20)   **Absolute Adjective Axiom (AAA):** For all $X \in \mathcal{P}(D)$ and $a_1 \in X$, $[\![Q_1(a_1)]\!]^c_{M,g,X} = 1$ iff $[\![Q_1(a_1)]\!]^c_{M,g,D} = 1$.

(21)   **Tolerant No Skipping (T-NS):** For an AA Q_1, $X \in \mathcal{P}(D)$ and $a_1, a_2 \in X$, if $a_1 \sim^X_{Q_1} a_2$ and there is some $a_3 \in X$ such that $[\![a_1 \geq_{Q_1} a_3]\!]^t_{M,g,X} = 1$ and $[\![a_3 \geq_{Q_1} a_2]\!]^t_{M,g,X} = 1$, then $a_1 \sim^X_{Q_1} a_3$.

(22)   **Granularity 1 (G1):** For an AA Q_1, $X \in \mathcal{P}(D)$, and $a_1, a_2 \in X$, if $a_1 \sim^X_{Q_1} a_2$, then for all $X' \in \mathcal{P}(D) : X \subseteq X'$, $a_1 \sim^{X'}_{Q_1} a_2$.

(23)   **Granularity 2 (G2):** For an AA Q_1, $X, X' \in \mathcal{P}(D)$, and $a_1, a_2 \in X$, if $X \subset X'$ and $a_1 \not\sim^X_{Q_1} a_2$ and $a_1 \sim^{X'}_{Q_1} a_2$, then $\exists a_3 \in X' - X : a_1 \not\sim^{X'}_{Q_1} a_3$.

(24)   **Minimal Difference (MD):** For an AA Q_1 and $a_1, a_2 \in D$, if $[\![a_1 >_{Q_1} a_2]\!]^c_{M,g,X} = 1$, then $a_1 \not\sim^{\{x,y\}}_{Q_1} a_2$.

7.2 Proofs

Firstly, Minimal Difference ensures that classical absolute denotations are subsets of tolerant denotations:

Lemma 1. *Tolerant Subset.* *If $Q \in AA$, then, for all $X \subseteq D$, $a_1, a_2 \in D$, if $[\![a_1 >_Q a_2]\!]^c_{M,g,X}$, then $[\![a_1 >_Q a_2]\!]^t_{M,g,X}$.*

Proof. Suppose $[\![a_1 >_Q a_2]\!]^c_{M,g,X}$. Then, by definition 20, there is some $X' \subseteq D$ such that $[\![Q(a_1)]\!]^c_{M,g,X'} = 1$ and $[\![Q(a_2)]\!]^c_{M,g,X'} = 0$. Now consider $\{a_1, a_2\}$. By downward difference, $[\![Q(a_1)]\!]^c_{M,g,\{a_1,a_2\}} = 1$ and $[\![Q(a_2)]\!]^c_{M,g,\{a_1,a_2\}} = 0$. By the definition of $[\![]\!]^t$, $[\![Q(a_1)]\!]^t_{M,g,\{a_1,a_2\}} = 1$. Furthermore, by Minimal Difference, $a_1 \not\sim^{\{a_1,a_2\}}_Q a_2$. So $[\![Q(a_2)]\!]^t_{M,g,\{a_1,a_2\}} = 0$. By definition 21, $[\![a_1 >_Q a_2]\!]^t_{M,g,X}$. $\qquad \square$

Secondly, with only T-No Skipping, we can prove that a version of van Bentham's No Reversal holds at the tolerant level.

Lemma 2. *Tolerant No Reversal (T-NR):* *For $X \subseteq D$, and $a_1, a_2 \in D$ if $[\![Q(a_1)]\!]^t_{M,g,X} = 1$ and $[\![Q(a_2)]\!]^t_{M,g,X} = 0$, then there is no $X' \subseteq D$ such that $[\![Q(a_2)]\!]^t_{M,g,X'} = 1$ and $[\![Q(a_1)]\!]^t_{M,g,X'} = 0$.*

Proof. Suppose $[\![Q(a_1)]\!]^t_{M,g,X} = 1$ and $[\![Q(a_2)]\!]^t_{M,g,X} = 0$. Suppose, for a contradiction that there is an $X' \subseteq D$ such that $[\![Q(a_2)]\!]^t_{M,g,X'} = 1$ and $[\![Q(a_1)]\!]^t_{M,g,X'} = 0$. Therefore, $[\![a_1 >_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_2 >_Q a_1]\!]^t_{M,g,X} = 1$. Furthermore, by assumption and definition 21, there is some $a_3 \sim^X_Q a_1$ such that $[\![Q(a_3)]\!]^c_{M,g,X} = 1$, and $a_3 \not\sim^X_Q a_2$. Since $[\![Q(a_1)]\!]^t_{M,g,X'} = 0$, by the AAA, $[\![Q(a_1)]\!]^c_{M,g,X'} = 0$.

---

[13] **Tolerantly greater than or equal**. ($\geq^t$) For an interpretation $[\![\cdot]\!]_{M,g,X}$, a predicate P, $a_1, a_2 \in D$, $a_1 \geq^t_P a_2$ iff $[\![a_1 >_P a_2]\!]^t_{M,g,X} = 1$ or $a_1 \approx^t_P a_2$.

So $[\![a_3 >_Q a_2]\!]^c_{M,g,X} = 1$. By lemma 1, $[\![a_3 >_Q a_2]\!]^t_{M,g,X} = 1$, and so $[\![a_3 >_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_2 >_Q a_1]\!]^t_{M,g,X} = 1$. Since $a_3 \sim^X_Q a_1$, by No Skipping, $a_3 \sim^X_Q a_2$. $\perp$ $\square$

Using the complete axiom set {NS, G1, G2, MD}, we can show that, for all $Q \in AA$, the tolerant comparative ($>^t_Q$) is a strict weak order.

Lemma 3. *Irreflexivity*. For all $X \subseteq D$ and $a_1 \in D$, $[\![a_1 >_Q a_1]\!]^t_{M,g,X} = 0$.

Proof. Since it is impossible, for any $X \subseteq D$, for an element to be both in $[\![Q]\!]^t_{M,g,X}$ and not in $[\![Q]\!]^t_{M,g,X}$, by definition 21, $>^t_Q$ is irreflexive. $\square$

Lemma 4. *Transitivity*. For all $X \subseteq D$ and $a_1, a_2, a_3 \in D$, if $[\![a_1 >_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_2 >_Q a_3]\!]^t_{M,g,X} = 1$, then $[\![a_1 >_Q a_3]\!]^t_{M,g,X} = 1$.

Proof. Suppose $[\![a_1 >_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_2 >_Q a_3]\!]^t_{M,g,X} = 1$ to show that $[\![a_1 >_Q a_3]\!]^t_{M,g,X} = 1$. Then there is some $X' \subseteq D$ such that $[\![Q(a_1)]\!]^t_{M,g,X'} = 1$ and $[\![Q(a_2)]\!]^t_{M,g,X'} = 0$. Thus, there is some $a_4 \in X' : [\![Q(a_4)]\!]^c_{M,g,X'} = 1$, and $a_4 \sim^{X'}_Q a_1$. Now consider $X' \cup \{a_3\}$. By the AAA and the assumption that $[\![a_1 >_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_2 >_Q a_3]\!]^t_{M,g,X} = 1$, $[\![Q(a_2)]\!]^c_{M,g,X\cup\{a_3\}} = 0$ and $[\![Q(a_3)]\!]^c_{M,g,X'\cup\{a_3\}} = 0$.
Case 1: $X' \cup \{a_3\} = X'$. Since $[\![Q(a_3)]\!]^t_{M,g,X'} = 1$ and $[\![Q(a_2)]\!]^t_{M,g,X'} = 0$, by theorem 2, $[\![Q(a_3)]\!]^t_{M,g,X'} = 0$, and $[\![a_1 >_Q a_3]\!]^t_{M,g,X} = 1$. ✓ **Case 2:** $X' \subset X' \cup \{a_3\}$. Since $X' \subset X' \cup \{a_3\}$ and $a_4 \sim^X_Q a_1$, by G1, $a_4 \sim^{X'\cup\{a_3\}}_Q a_1$. By the AAA, $[\![Q(a_4)]\!]^c_{M,g,X'\cup\{a_3\}} = 1$. So $[\![Q(a_1)]\!]^t_{M,g,X\cup\{a_3\}} = 1$. Suppose, for a contradiction that $[\![Q(a_3)]\!]^t_{M,g,X'\cup\{a_3\}} = 1$. Then there is some $a_5 \in X' \cup \{a_3\} : [\![Q(a_5)]\!]^c_{M,g,X'\cup\{a_3\}} = 1$ and $\mathbf{a_5} \sim^{\mathbf{X'\cup\{a_3\}}}_{\mathbf{Q}} \mathbf{a_3}$. By assumption and since $[\![Q(a_2)]\!]^c_{M,g,X'}$, by MD, $[\![a_5 >_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_2 >_Q a_3]\!]^t_{M,g,X} = 1$. So by T- No Skipping, $a_5 \sim^{X'\cup\{a_3\}}_Q a_2$. Since $[\![Q(a_2)]\!]^t_{M,g,X'} = 0$, $a_5 \not\sim^{X'}_Q a_2$. So by G2, since $X' \cup \{a_3\} - X' = \{a_3\}$, $\mathbf{a_5} \not\sim^{\mathbf{X'\cup\{a_3\}}}_{\mathbf{Q}} \mathbf{a_3}$. $\perp$. So $[\![Q(a_3)]\!]^t_{M,g,X'\cup\{a_3\}} = 0$, and $[\![a_1 >_Q a_3]\!]^t_{M,g,X} = 1$. ✓ $\square$

Lemma 5. *Almost Connectedness*. For all $X \subseteq D$ and $a_1, a_2 \in D$, if $[\![a_1 >_Q a_2]\!]^t_{M,g,X} = 1$ then for all $a_3 \in D$, either $[\![a_1 >_Q a_3]\!]^t_{M,g,X} = 1$ or $[\![a_3 >_Q a_2]\!]^t_{M,g,X} = 1$.

Proof. Let $[\![a_1 >_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_3 >_Q a_2]\!]^t_{M,g,X} = 0$ to show $[\![a_1 >_Q a_3]\!]^t_{M,g,X} = 1$.
Case 1: $[\![Q(a_1)]\!]^c_{Mg,D} = 1$. Since $[\![a_1 >_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_3 >_Q a_2]\!]^t_{M,g,X} = 0$, $[\![Q(a_3)]\!]^c_{M,g,D} = 0$. So $[\![a_1 >_Q a_3]\!]^c_{M,g,X} = 1$, and, by lemma 1, $[\![a_1 >_Q a_3]\!]^t_{M,g,X} = 1$. ✓ **Case 2:** $[\![Q(a_1)]\!]^c_{Mg,D} = 0$. Since $[\![a_1 >_Q a_2]\!]^t_{M,g,X} = 1$, there is some $X' \subseteq D$ such that $[\![Q(a_1)]\!]^t_{M,g,X'} = 1$ and $[\![Q(a_2)]\!]^t_{M,g,X'} = 0$. So there is some $a_4 \in X' : [\![Q(a_4)]\!]^c_{M,g,X'} = 1$ and $a_4 \sim^{X'}_Q a_1$. Now consider $X' \cup \{a_3\}$. Since

$[\![a_3 >_Q a_2]\!]^t_{M,g,X} = 0$, $[\![Q(a_1)]\!]^c_{M,g,X'\cup\{a_3\}} = 0$ and $[\![Q(a_2)]\!]^c_{M,g,X'\cup\{a_3\}} = 0$ and $[\![Q(a_3)]\!]^c_{M,g,X'\cup\{a_3\}} = 0$. Since $a_4 \sim^{X'}_Q a_1$, by G1, $a_4 \sim^{X'\cup\{a_3\}}_Q a_1$ and by the AAA, $[\![Q(a_4)]\!]^c_{M,g,X'\cup\{a_3\}} = 1$. So, by definition 21, $[\![Q(a_1)]\!]^t_{M,g,X'\cup\{a_3\}} = 1$. Now suppose for a contradiction that $[\![Q(a_3)]\!]^t_{M,g,X'\cup\{a_3\}} = 1$. Then there is some $a_5 \in X' \cup \{a_3\} : [\![Q(a_5)]\!]^c_{M,g,X'\cup\{a_3\}} = 1$ and $\mathbf{a_5} \sim^{\mathbf{X'\cup\{a_3\}}}_{\mathbf{Q}} \mathbf{a_3}$. Since $[\![Q(a_5)]\!]^c_{X'\cup\{a_3\}} = 1$ and $[\![Q(a_2)]\!]^c_{X'\cup\{a_3\}} = 0$, $[\![a_5 >_Q a_2]\!]^c_{M,g,X} = 1$; so by lemma 1, $[\![a_5 >_Q a_2]\!]^t_{M,g,X} = 1$. Furthermore, since, by assumption, $[\![a_3 >_Q a_2]\!]^t_{M,g,X} = 0$, $[\![a_2 \geq_Q a_3]\!]^t_{M,g,X} = 1$. Since $[\![a_5 \geq_Q a_2]\!]^t_{M,g,X} = 1$ and $[\![a_2 \geq_Q a_3]\!]^t_{M,g,X} = 1$, and $a_5 \sim^{X'\cup\{a_3\}}_Q a_3$, by Tolerant No Skipping, $a_5 \sim^{X'\cup\{a_3\}}_Q a_2$. However, since $[\![Q(a_2)]\!]^t_{M,g,X'} = 0$, and by the AAA, $[\![Q(a_5)]\!]^c_{M,g,X'} = 1$, $a_5 \not\sim^{X'}_Q a_2$. Since $X' \subset X' \cup \{a_3\}$ and $a_5 \sim^{X'\cup\{a_3\}}_Q a_2$, by G2, there is some $a_6 \in X' \cup \{a_3\} - X'$ such that $a_6 \not\sim^{X'\cup\{a_3\}}_Q a_3$. Since $X' \cup \{a_3\} - X' = \{a_3\}$, $\mathbf{a_5} \not\sim^{\mathbf{X'\cup\{a_3\}}}_{\mathbf{Q}} \mathbf{a_3}$. $\bot$ So $[\![Q(a_3)]\!]^t_{M,g,X'\cup\{a_3\}} = 0$ and $[\![a_1 >_Q a_3]\!]^t_{M,g,X} = 1$. ✓ □

We can now prove the main theorem of the paper:

Theorem 3. *If Q is an absolute adjective, $<^t_Q$ is a strict weak order.*

Proof. Immediate from lemmas 3, 4 and 5. □