Introduction to Quantification in Natural Language

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1 What is the class about?

The main goal of this class is to provide a brief introduction to the phenomenon of quantification in natural language and its analysis within Generalized Quantifier (GQ) theory.

There are many ways one could teach this kind of course!

- The particular focus will be on the basic semantic characteristics of quantificational expressions and their realizations across syntactic domains.
  - What are the semantic and syntactic properties of DP quantifiers in human languages?
  - How do we know that a particular linguistic expression is a quantifier?
- If we have time: What are the properties of quantifiers in the adjectival and verbal domains?

1.1 Plan

Class 1: Introduction to GQ theory.
- Introduction to model-theoretic semantics.
- Initial motivations for GQ theory.

Class 2: What GQ theory is good for.
- Empirical application 1: Negative polarity licensing.
- Empirical application 2: Existential constructions.

Class 3: Transitive predicates and scope.
- The interpretation of quantified noun phrases in object position.
- The derivation of scope ambiguities.

Class 4: Some things that may not be quantifiers.
- The scope properties of indefinite DPs.
- Universal modifiers (ex: all).

Class 5: Quantification in the adjectival and verbal domains. (See also Roumi’s class)
- Comparatives as quantifiers over degrees or comparison classes.
- Event quantification.


2 Basic Formal Semantics

Semantics is the branch of linguistics that investigates the meaning of simple and complex expressions of natural languages.

**Question:** How do we interpret sentences of English (or Polish, French, Tongan . . .)?

![Figure 1: Lisa is vegetarian.](image)

Perspectives adopted in this course: **Truth-conditional semantics** and its closely related cousin: **Model-theoretic semantics**.

**Truth conditional semantics (Tarski, Davidson):** To know the meaning of an assertion is to know the conditions under which the assertion is true.

\[(1) \quad \text{Lisa is vegetarian. is true iff Lisa has the property of being a vegetarian (doesn’t eat meat etc.)} \]

- In this class, we will be interested in studying *entailments* between sentences with(out) quantified DPs. (An entailment = a certain kind of implicational relation between sentences)
- To study entailments, we will adopt a model-theoretic semantic framework.

**Model-theoretic semantics (Montague):** A truth-conditional theory in which the truth of an expression is evaluated with relative to a model.

Informally (for now):

\[(2) \quad \text{A model } M \text{ is the combination of a domain/universe } (D_M) \text{ and an interpretation function } ([\cdot]_M).\]

- When \([\cdot]_M \text{ applies to complete assertions, it yields a truth value, which is determined by the features of the model.}\]

\[(3) \quad [\text{Lisa is vegetarian}]_M = \text{true iff Lisa is a vegetarian in } M.\]

a. \(\text{Lisa is vegetarian. is true w.r.t. a model } M \text{ iff Lisa is a vegetarian in } M.\)

**Question:** How do we get from a particular linguistic expression (i.e. *Lisa is vegetarian.*) to its truth conditions?

**Hypothesis 1:** Memorization.
• We know we have to learn some form-meaning associations (new words, idioms etc.).

(4) John [kicked the bucket].

• We learn that the whole sentence Lisa is vegetarian is assigned the truth conditions true iff Lisa doesn’t eat meat.

Memorization is not good enough:

1. **Productivity (Frege’s observation):** We can understand sentences that we have never heard before.

   • (Semantic version of) Chomsky’s observation: We can understand an infinite number of sentences that we have never heard before.

(5) An enormous pink flamingo just ate the President of Poland’s old hat.

(6) a. An enormous pink flamingo just ate the President of Poland’s very old hat.
   b. An enormous pink flamingo just ate the President of Poland’s very, very old hat.
   c. An enormous pink flamingo just ate the President of Poland’s very, very, very old hat.
   And so on...

2. **Systematicity (Pāṇini’s observation):** Substitution of sub-sentential parts changes meaning in a predictable way.

(7) a. Bring me a glass of wine.
   b. Bring me a beer.

**Hypothesis 2:** Compositionality.

• We learn 1) the meaning of a finite number of syntactically atomic parts (words/morphemes) and 2) rules for interpreting infinitely many syntactic structures containing them. (cf. productivity)

• The meaning of a syntactically complex expression is a function of the meaning of its atomic parts and these interpretation rules. (cf. systematicity)

2.1 **Compositional Interpretation of Simple Sentences**

Our interpretation function ([·]) has two parts:

1. It first assigns a semantic value (called a denotation) to syntactically basic constituents (words).

2. [·] is extended to assign semantic values to syntactically complex constituents.

We can now adopt a more formal definition of a model:
**Definition 2.1  Model.** A model $M$ is a pair $\langle D_M, \cdot \rangle_M$, where $D_M$ is a set of individuals and $\cdot$ is a function from a finite set of syntactically atomic constituents to appropriate semantic values.

What are the possible semantic values of words of different syntactic categories?

(8) a. Lisa is vegetarian.
   b. Bart is vegetarian.
   c. Homer is vegetarian.

- Simplifying assumption: *is* is semantically null.
- Two syntactically atomic constituents in (8-a): *Lisa* and *vegetarian*.

**Simple proposals:**

- Proper names are assigned individuals in the domain by $\cdot$.
  (*Proper names denote individuals.*)

- Adjectives are assigned properties: sets of individuals by $\cdot$.
  (*Adjectives denote sets of individuals.*)

Now we need a rule for interpreting full sentences:

(9) **Interpretation of Complex Expressions:**

If $\alpha$ has the form $\beta \gamma$, then $\alpha = \text{True}$ iff $\beta \in \gamma$.

In other words:

- A sentence with a proper name subject and an intransitive predicate is true (in a model) iff the individual denoted by the subject has the property denoted by the predicate (in the model).

(10) $[\text{DP Pred}]_M = \text{True}$ iff $[\text{DP}]_M \in [\text{Pred}]_M$.

(11) An example model $M$:

   a. $D_M = \{\text{Lisa, Bart, Homer}\}$
   b. $[\text{Lisa}]_M = \text{Lisa}$; $[\text{Bart}]_M = \text{Bart}$; $[\text{Homer}]_M = \text{Homer}$.
   c. $[\text{(is) vegetarian}]_M = \{\text{Lisa}\}$.
   d. $[\text{drinks}]_M = \{\text{Homer}\}$.

(12) Are the sentences *Lisa is vegetarian* and *Bart is vegetarian* true in $M$?

\[
\begin{align*}
[\text{Lisa is vegetarian}]_M &= \text{True} & (\text{Lisa } \in \{\text{Lisa}\}) \\
[\text{Lisa}]_M &= \text{Lisa} & [\text{(is) vegetarian}]_M = \{\text{Lisa}\} \\
[\text{Bart is vegetarian}]_M &= \text{False} & (\text{Bart } \notin \{\text{Lisa}\}) \\
[\text{Bart}]_M &= \text{Bart} & [\text{(is) vegetarian}]_M = \{\text{Lisa}\}
\end{align*}
\]
Exercises

a. \([\text{Homer is vegetarian}]_M = ?\)
b. \([\text{Homer drinks}]_M = ?\)
c. \([\text{Bart drinks}]_M = ?\)

2.1.1 Summary

1. We interpret complex sentences compositionally in a model (a pair consisting of a domain/universe and an interpretation function).

2. We assumed that proper name subjects denote individuals in the model and predicates denote properties (sets of individuals).

3. We have one rule for interpreting full sentences composed of a subject and a predicate:

   \[
   [\text{DP Pred}]_M = \text{True iff } [\text{DP}]_M \in [\text{Pred}]_M.
   \]

2.2 What about quantified DPs?

This is too simple! I have only shown you how to interpret sentences with singular proper name subjects and intransitive predicates (Lisa is vegetarian; Homer drinks...).

- There are lots of other kinds of DP subjects in English (and other languages)!

- How do we calculate the truth conditions and truth values of sentences with quantified subjects (15)?

(15)  
a. \([\text{Everyone is vegetarian}] = \text{True iff } ?\)
b. \([\text{No one is vegetarian}] = \text{True iff } ?\)
c. \([\text{Someone is vegetarian}] = \text{True iff } ?\)

We know what the denotation of vegetarian is: the set of vegetarians (For a model \(M\), \([\text{vegetarian}]_M = \{x : x \text{ is a vegetarian in } M\}\)).

(16)  
a. \([\text{Everyone}]_M = ?\)
b. \([\text{No one}]_M = ?\)
c. \([\text{Someone}]_M = ?\)

Hypothesis 1: Everyone/no one/someone be individual denoting like Lisa?

- Argument against hypothesis 1: Which individual does everyone denote? Which individual does no one denote?

Hypothesis 2: The meaning of everyone/someone etc. is more similar to that of \(\forall\) and \(\exists\) in first order predicate logic.

- We can make universal and existential statements in predicate logic.

- We have a way of compositionally interpreting them.

Maybe we can apply the semantic theory of predicate logic to the interpretation of DPs like everyone and no one...
3 Quantifiers in Predicate Logic

Question: How do you make particular statements about individuals in predicate logic?

• How do you say Lisa is vegetarian in predicate logic?

Answer: You concatenate a predicate symbol with a constant (similar to in English).

• The semantic denotation of a formula like in (17) is calculated in a very similar manner to the one described in section 2 for Lisa is vegetarian.

\( [V(l)]_M = \text{True iff } [l]_M \in [V]_M. \)

Question: How do you express general statements in predicate logic?

• How do you say (18) in predicate logic?

(18) a. Everyone is vegetarian.
    b. Someone is vegetarian.

Answer: In a very different way from English!

• The English DP everyone is split across three syntactic constituents: \( \forall \) and two occurrences of \( x \).

(19) a. \( [\forall x (V(x))]_M = \text{True iff for all individuals } x, x \in [V]_M. \)
    b. \( [\exists x (V(x))]_M = \text{True iff there is some individual } x \text{ such that } x \in [V]_M. \)

• Furthermore, in predicate logic, quantifier symbols like \( \forall \) are syncategorematic: they have no meaning outside complete sentences.

• Although, in predicate logic, we have a compositional way to assign meanings to sentences that have quantifier symbols in them, the quantifier symbols themselves are meaningless.

• \( [\forall] = ? \)

Question: How do you say No one is vegetarian?

Answer: You separate no one into a quantifier symbol \( \exists/\forall \), a sentential negation symbol \( \neg \) and two occurrences of a variable.

• Although there is some controversy over the syntactic decomposition of negative quantifiers in natural languages, the syntax of negative quantified statements in predicate logic is still very different from the syntax of such sentences in English.

(20) a. \( \forall x (\neg (V(x))) \)
    b. \( \neg (\exists x (V(x))) \)
3.1 Summary

Good thing: We have a way of compositionally assigning truth values to sentences with quantifiers in predicate logic.

- But the syntax of first order predicate logic is so different from the syntax of any natural language that it is unclear to what extent we can directly apply its semantic theory to English.
- We still have no idea what the denotation of everyone in English could be, since predicate logic does not even have constituents like everyone.

Conclusion:

We need some way of integrating the semantics of quantifiers within predicate logic with the syntax of quantified DPs in English and other natural languages.

Enter Generalized Quantifier Theory!

4 Generalized Quantifier Theory

Generalized Quantifier (GQ) Theory is a framework for analyzing the semantics of natural language DPs that allows for a uniform and categorematic treatment of quantified noun phrases.

- Initial work by Mostowski (1957).

Proposal:

The truth conditions of natural language sentences with quantified DP subjects can be framed as relations between properties.

(21) \([\text{Everyone is vegetarian.}]_M = \text{True iff } D_M \subseteq [\text{vegetarian}]_M.\]
   a. Everyone is vegetarian is true (in a model) iff the set of all the individuals in the model is a subset of the set of vegetarians in the model.
   b. Everyone is vegetarian is true (in a model) iff for all individuals \(x\) in the domain, \(x\) is vegetarian.

(22) \([\text{Someone is vegetarian.}]_M = \text{True iff } D_M \cap [\text{vegetarian}]_M \neq \emptyset.\]
   a. Someone is vegetarian is true (in a model) iff the intersection of the domain of the model and the set of vegetarians in the model is non-empty.
   b. Someone is vegetarian is true (in a model) iff there is at least some individual in the model that is vegetarian.

(23) \([\text{No one is vegetarian.}]_M = \text{True iff } D_M \cap [\text{vegetarian}]_M = \emptyset.\]
   a. No one is vegetarian is true (in a model) iff the intersection of the domain of the model and the set of vegetarians in the model is empty.
   b. No one is vegetarian is true (in a model) iff there is no individual in the model that is vegetarian.
Proposal for DP denotations:

Subject DPs denote properties of properties (=def generalized quantifiers).

If we analyze properties as sets of individuals, then subject DPs denote sets of sets of individuals.

(24) Quantified DP denotations

a. \[ \text{Everyone}_M = \{X : D_M \subseteq X\} \]
b. \[ \text{Someone}_M = \{X : D_M \cap X \neq \emptyset\} \]
c. \[ \text{No one}_M = \{X : D_M \cap X = \emptyset\} \]

How do quantified subjects combine with predicates?

- Intuitively, we can see that our interpretation rule from section 2 (25) will not work.

(25) \[ [\text{DP Pred}]_M = \text{True iff } [\text{DP}]_M \in [\text{Pred}]_M. \]

- Quantified DPs denote families of sets, while predicates simply denote sets.
- A quantified DP denotation will never be a member of a predicate denotation.
- But the reverse is possible: the quantified subject DP is itself a predicate of properties.

(26) Interpretation of sentences with quantified noun phrases

For a quantified subject DP: \[[\text{DP Pred}]_M = \text{True iff } [\text{Pred}]_M \in [\text{DP}]_M.\]

(27) Recall our example model \(M\):

a. \(D_M = \{\text{Lisa, Bart, Homer}\}\)
b. \([\text{Lisa}]_M = \text{Lisa}; [\text{Bart}]_M = \text{Bart}; [\text{Homer}]_M = \text{Homer}.\)
c. \([\text{(is) vegetarian}]_M = \{\text{Lisa}\}.\)
d. \([\text{drinks}]_M = \{\text{Homer}\}.\)

(28) Are the sentences Everyone is vegetarian and Someone is vegetarian true in \(M\)?

\[[\text{Everyone is vegetarian}]_M = \text{False } \{\text{Lisa}\} \notin \{\{\text{Lisa, Bart, Homer}\}\}\]

\[[\text{Everyone}]_M = \{\{\text{Lisa, Bart, Homer}\}\} \quad [\text{(is) vegetarian}]_M = \{\text{Lisa}\}\]

\[[\text{Someone is vegetarian}]_M = \text{True } \{\text{Lisa}\} \in \{\{\text{Lisa}\}, \{\text{Bart}\}...\}\]

\[[\text{Someone}]_M = \{\{\text{Lisa}\}, \{\text{Bart}\}, \{\text{Lisa, Bart}\}...\} \quad [\text{(is) vegetarian}]_M = \{\text{Lisa}\}\]

4.0.1 Summary

- Within the GQ approach, quantified subject DPs denote properties of properties.
  - With this analysis, we can arrive at a compositional interpretation of English sentences like Everyone is vegetarian and No one is vegetarian.
• Now we have two distinct interpretation rules: one for when the sentence has a proper name subject (25) and one for when the sentence has a quantified subject (26).

Do we really need two interpretation rules? Or can we do with one?

4.1 Montagovian Individuals

In analogy to the treatment of constants in predicate logic, we analyzed singular proper names as directly denoting individuals.

(29) \([\text{Lisa}]_M = \text{Lisa}\).

• A sentence with a proper name DP subject is true just in case the individual denoted by the subject is included in the set denoted by the predicate.

Question: Can we assign a denotation to Lisa that is of the same type as everyone?

Montague’s Answer: Yes! Proper names like Lisa also denote sets of properties, namely those properties that hold of Lisa.

• Proper names denote Montagovian individuals (a special kind of generalized quantifier).

Definition 4.1 Montagovian individual. Let \(M\) be a model. For all \(a \in D_M\), let \(I_a\) (read: the montagovian individual on \(a\)) be the following family of properties:

(30) \(I_a = \text{def} \{X : a \in X\}\)

(31) a. \([\text{Lisa}]_M = \{X : \text{Lisa} \in X \text{ in } M\}\)
b. \([\text{Bart}]_M = \{X : \text{Bart} \in X \text{ in } M\}\)

Now we can use our second interpretation rule for all DPs:

(32) Interpretation of Intransitive Sentences
\([\text{DP Pred}]_M = \text{True iff } [\text{Pred}]_M \in [\text{DP}]_M\).

We derive the same truth conditions as with our first rule.

\([\text{Lisa is vegetarian}]_M = \text{True} \quad \{\{\text{Lisa}\} \in \{X : \text{Lisa} \in X\}\}\)

\([\text{Lisa}]_M = \{X : \text{Lisa} \in X\} \quad [(\text{is vegetarian})]_M = \{\text{Lisa}\}\)

\([\text{Bart is vegetarian}]_M = \text{False} \quad \{\{\text{Lisa}\} \notin \{X : \text{Bart} \in X\}\}\)

\([\text{Bart}]_M = \{X : \text{Bart} \in X\} \quad [(\text{is vegetarian})]_M = \{\text{Lisa}\}\)

Conclusion: By treating all subject DPs as generalized quantifiers, we can arrive at:
1. A categorematic analysis of the semantic denotation of quantified noun phrases.

2. A unified treatment of the compositional semantics of proper name subjects and quantified subjects.

5 Next Class

1. Below the DP level: what are determiner denotations?

   (33)   a. Every boy is vegetarian.
          b. No girl is vegetarian.

   (34)   a. \texttt{every} = ?
          b. \texttt{no} = ?

2. How the abstract properties of determiners can help us explain syntactic and semantic patterns in natural languages.

   (35) NPI Licensing
        a. *Everyone at all came to the party.
        b. No one at all came to the party.

   (36) Existential Constructions
        a. There is a man in the garden.
        b. *There is every man in the garden.

References


